

8 - Solutions

1. How many ways can letters in the word 'NEEDLESSLY' be rearranged?

Answer: There are 10 letters, 3 E's, 2 L's, 2 S's so the answer is $\frac{10!}{3!2!2!}$.

2. In a horse race, how many possible outcomes are there for first, second, and third place if there are 27 horses racing?

Answer: $P(27, 3) = 27 \times 26 \times 25$.

3. A bag has 12 differently colored marbles. How many different color combos can be selected by simultaneously drawing three marbles from the bag?

Answer: $C(12, 3) = \frac{12!}{3!9!}$

4. Three different prizes will be awarded to 35 graduating students. If no student can receive more than one prize, in how many different ways could the prizes be awarded?

Answer: since the prizes are different, we use permutation instead of combinations: $P(35, 3) = 35 \times 34 \times 33$

5. How many 5-card hands have no red cards?

Answer: we choose 5 non-red cards from the 26 non-red cards in a standard deck of cards, so $C(26, 5)$.

6. How many 5-card hands have only heart cards?

Answer: we choose 5 heart cards from the 13 heart cards in a standard deck of cards, so $C(13, 5)$.

7. A school dance committee is to consist of 2 freshmen, 3 sophomores, 4 juniors, and 5 seniors. If 6 freshmen, 7 sophomores, 7 juniors, and 7 seniors are eligible to be on the committee, in how many ways can the committee be chosen?

Answer: $\binom{6}{2} \times \binom{7}{3} \times \binom{7}{4} \times \binom{7}{5}$

8. How many 8-bit strings have exactly two 1's? ("1100000", "01000100")

Answer: $\frac{8!}{2!6!}$

Non-Homework Problems.

9. How many 8-bit strings have at least three 1's?

Answer: It is easier to count the complement (how many 8-bit strings have at most two 1's). Overall there are $2^8 = 256$ many 8-bit strings. Of these, $C(8, 0) + C(8, 1) + C(8, 2) = 1 + 8 + 28 = 37$ of them have at most 2 1's. So the answer is $256 - 37 = 219$.

10.

- (a) How many arrangements of the letters in BALLYNALALLY are possible?
- (b) Of these arrangements, how many have all the L's together?

- (c) Of these arrangements, how many have all the letters in alphabetical order?

Answer (a): This word has 12 letters, 3 A's, 5 L's, 2 Y's so the answer is $\frac{12!}{3!5!2!}$

Answer (b): With all L's are grouped together like BALLYNALALLY \rightarrow BALLLLLYNAAY = BA(LLLLL)YNAAY, we can treat the L's as a single unit or character. So the problem reduces to counting the number of rearrangements of the letters in BALYNAAY which has 8 letters, 3 A's, 2 Y's so the answer is $\frac{8!}{3!2!}$

Answer (c): There is only one way to rearrange all letters in alphabetical order.

11. How many 5-card hands have exactly 2 hearts or 3 clubs?

Answer:

Let A be the set of 5-card hands with exactly 2 hearts. Then A has $\binom{13}{2} \times \binom{39}{3}$ elements.

Let B be the set of 5-card hands with exactly 3 clubs. Then A has $\binom{13}{3} \times \binom{39}{2}$ elements.

$A \cap B$ is then the set of 5-card hands with exactly 2 hearts and 3 clubs. This has $\binom{13}{2} \times \binom{13}{3}$ elements.

By inclusion-exclusion, $|A \cup B| = |A| + |B| - |A \cap B| = \binom{13}{2} \times \binom{39}{3} + \binom{13}{3} \times \binom{39}{2} - \binom{13}{2} \times \binom{13}{3}$.